The EUT Method for Stochastic Problems in Power System Analysis

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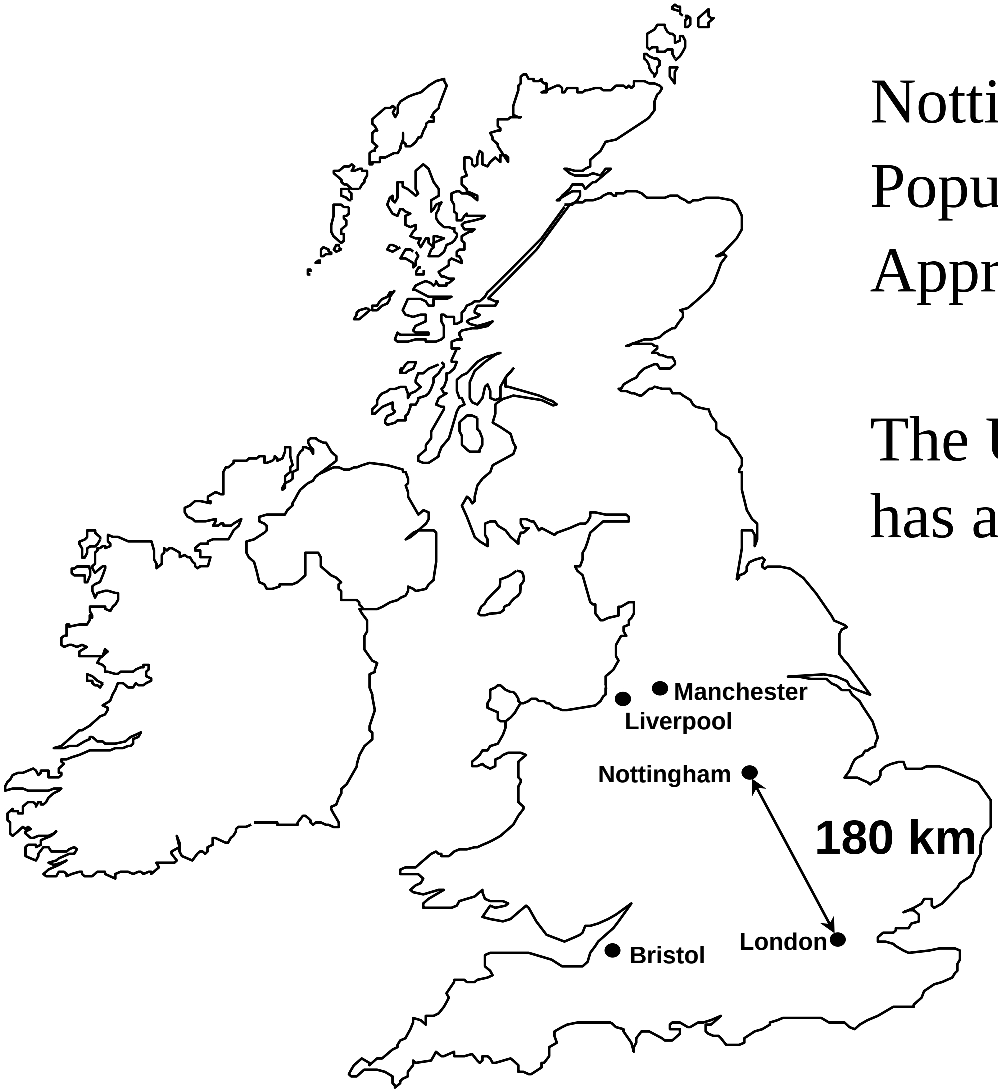


Nottingham
Population is
Approx. 300,000

The University of Nottingham
has approx. 30,000 students

The University of Nottingham

- Nottingham Civic college 1881
- Full University in 1948
- UK Russel Group University Ranked in the top 1% in the world
- 3 Campuses
  - – Nottingham, UK (33,000 Students)
  - – Malaysia (5000 Students)
  - – China (6000 Students)

Famous for

ROBIN
HOOD

Also Famous for

George Green

The George Green Institute for Electromagnetics Research

Established in 2004 and led by 8 academics:

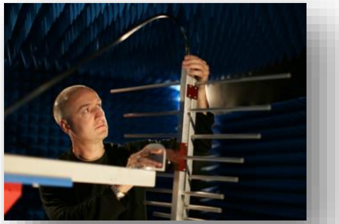
Dave Thomas (Director)

Trevor Benson

Slaweck Sujecki

Ana Vukovic

Angela Nothofer



Steve Greedy

Kristof Cools

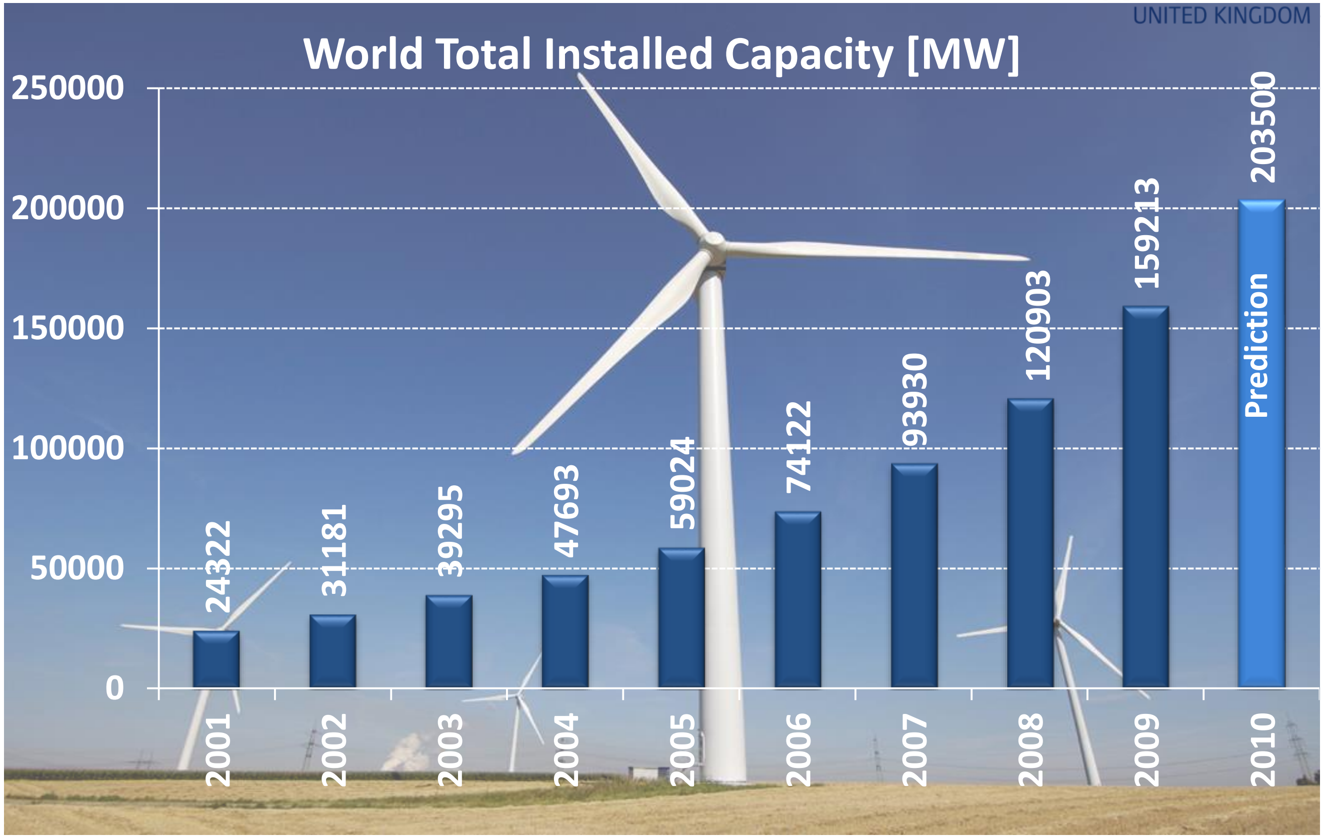
Phillip Sewell

Background

The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

Wind energy accounted for over 20% of world's total electricity installations in 2009

(Wind energy report 2010)



Over 15% of total world electricity generation to be from wind by 2020 (DOE, 2010)

Probabilistic load flow was introduced in 1974 to properly account for uncertainties in the network during load flow studies.

Probabilistic Modeling of Uncertainties

Some sources of uncertainties within the network.

- Load
- Generator
- Wind generator

- Wind data fitted to the *Weibull Distribution*

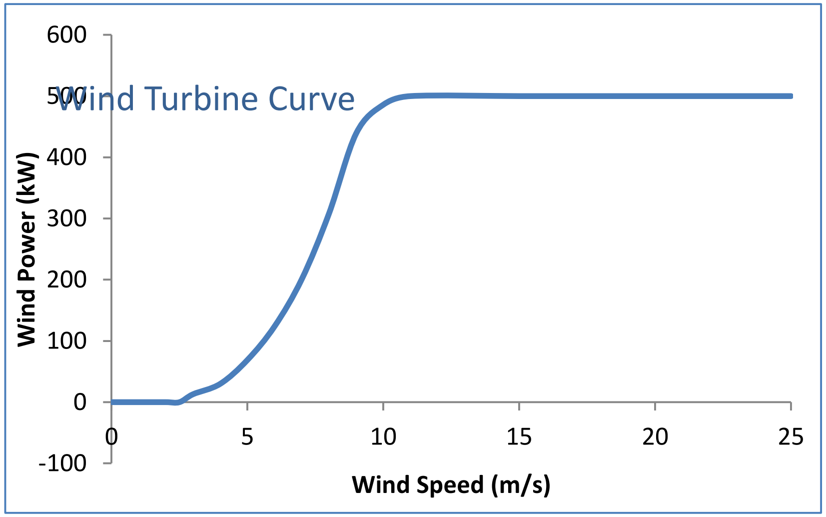
- Wind power obtained wind speed and turbine output curve.

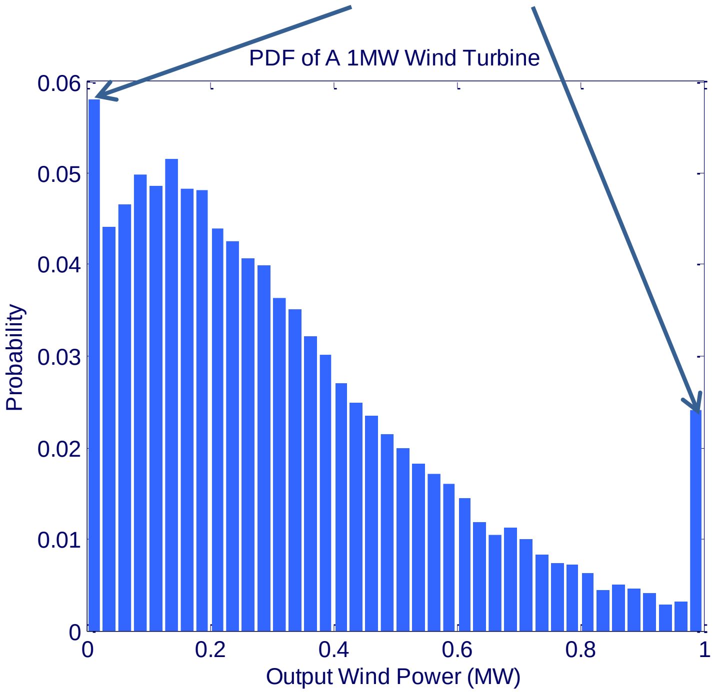
- Output wind power obtained as *Truncated Weibull-Degenerate Distribution.*

Wind Power Distribution

Wind Speed follows the Weibull Distribution with PDF

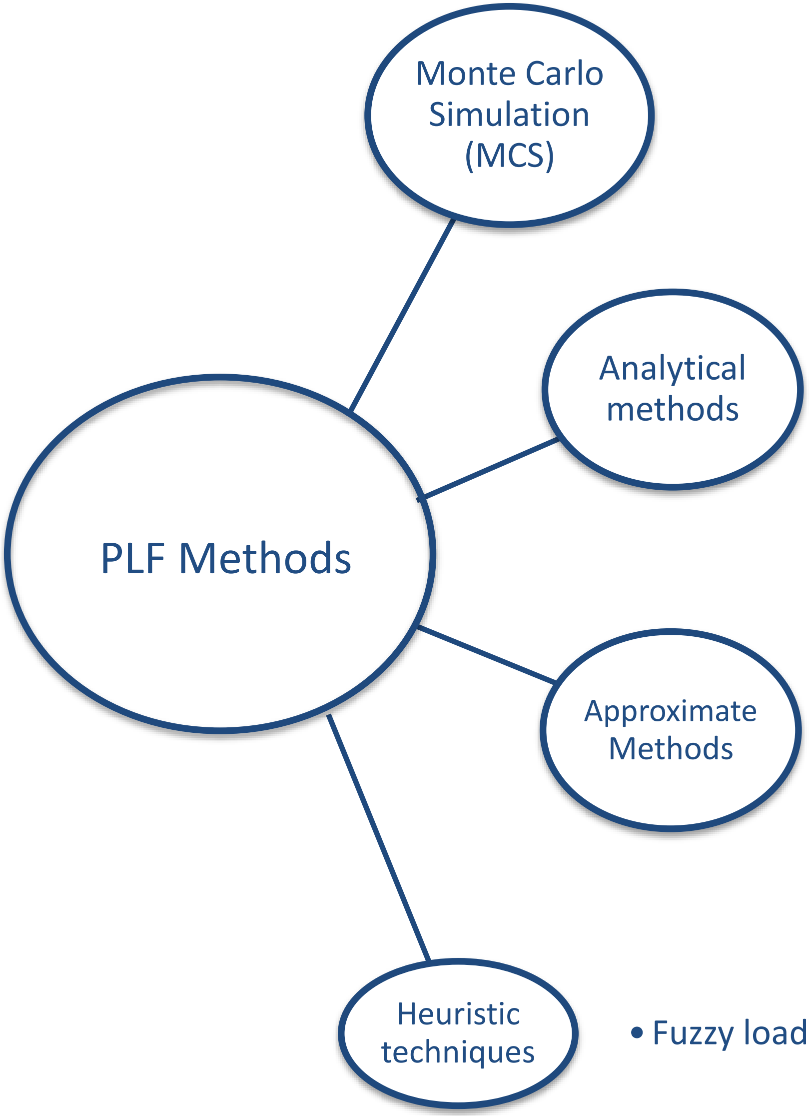
$$f(v) = \frac{\alpha}{\beta}\left(\frac{v-v_o}{\beta}\right)^{\alpha-1}\exp\left[-\left(\frac{v-v_o}{\beta}\right)^{\alpha}\right]$$




Active Points

$$f(P_w) = \begin{cases} \frac{\alpha}{\beta C_1}\left(\frac{P_w - C_1(vo - v_{ci})}{\beta C_1}\right)^{\alpha-1}\exp\left[-\left(\frac{P_w - C_1(vo - v_{ci})}{\beta C_1}\right)^{\alpha}\right] & 0 < P_w < P_r \\ \exp\left[-\left(\frac{v_r - v_o}{\beta}\right)^{\alpha}\right] - \exp\left[-\left(\frac{v_{co} - v_o}{\beta}\right)^{\alpha}\right] & P_w = P_r \end{cases} \qquad (4)$$

Existing PLF Methods



Monte Carlo Simulation (MCS)

Analytical methods

- Convolution method
- FFT technique
- Cumulant method

PLF Methods

Approximate Methods

- Point Estimate method (PEM)
- **Unscented Transforms (UT) method**

Heuristic techniques

- Fuzzy load flow

Probabilistic Modeling of Uncertainties

- Monte Carlo computationally very expensive
- UT greatly reduces number of simulations
- EUT extends the capability of the UT method

The Unscented Transforms (UT) Method

Works by approximating a continuous distribution function as a discrete distribution using deterministically chosen points such that both distributions have the same moments

$$E(\hat{u}^k) = \int \underbrace{\hat{u}^k w(\hat{u}) d\hat{u}}_{\textit{Continuous moment}} = \underbrace{\sum_i w_i S_i^k}_{\textit{Discrete moment}}$$

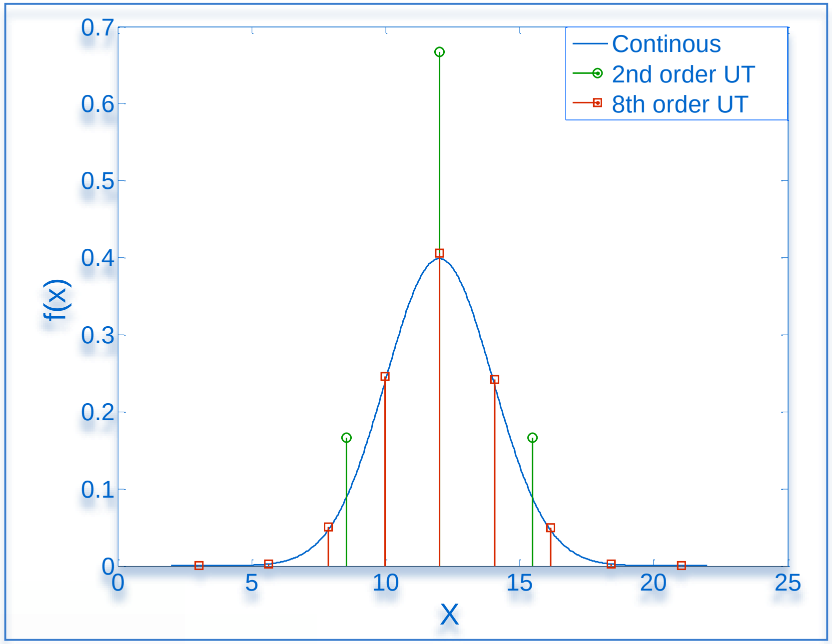


Illustration of continuous and discrete PDF

Conventional UT

- Based on Taylor's series expansion or some moment related method

×*Limited to a few orders of approximation*

×*Inaccurate and results in complex points for arbitrary measures like the Rayleigh distribution.*

UT as a Gaussian Quadrature Problem

- Integration of the UT equation is carried out using a quadrature technique.
- Desired sigma points (Si) correspond to the root of a polynomial orthogonal to weighting function.
- *No classical orthogonal polynomial associated with the Rayleigh distribution*
- Polynomial orthogonal to Rayleigh distribution is built from scratch.

Orthogonal Polynomials

- f_n and f_m are orthogonal if

$$\langle f_n, f_m \rangle = \int_a^b f_n(u) f_m(u) dW(u) = 0$$

where $(dW(u)=w(u)du)$

- *To derive orthogonal polynomials Moment method is* *extremely ill-conditioned for arbitrary measures.*

- *Discretization schemes such as the* STIELJES PROCEDURE *are better alternatives.*

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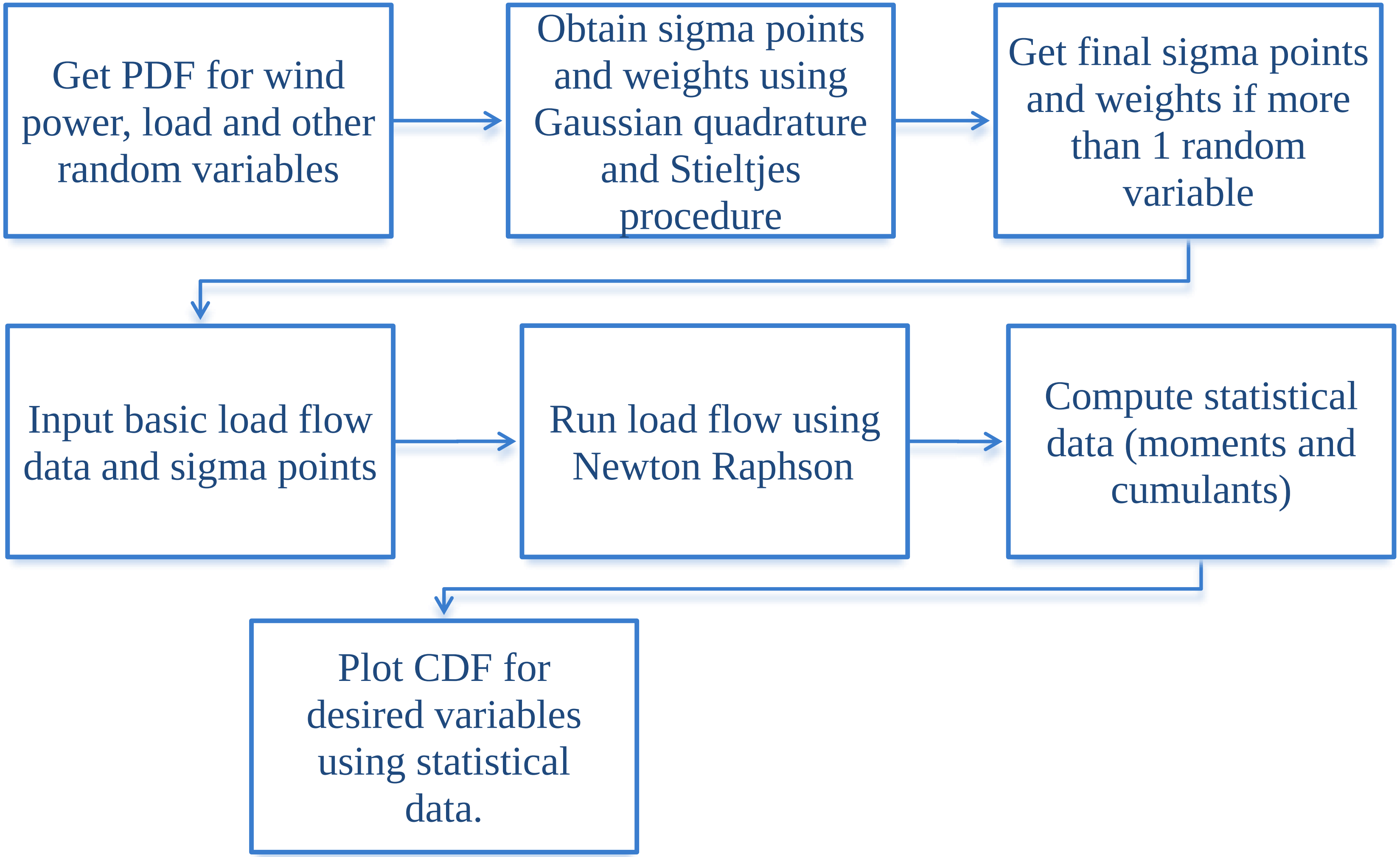
Implementation Procedure



Get PDF for wind power, load and other random variables

Obtain sigma points and weights using Gaussian quadrature and Stieltjes procedure

Get final sigma points and weights if more than 1 random variable

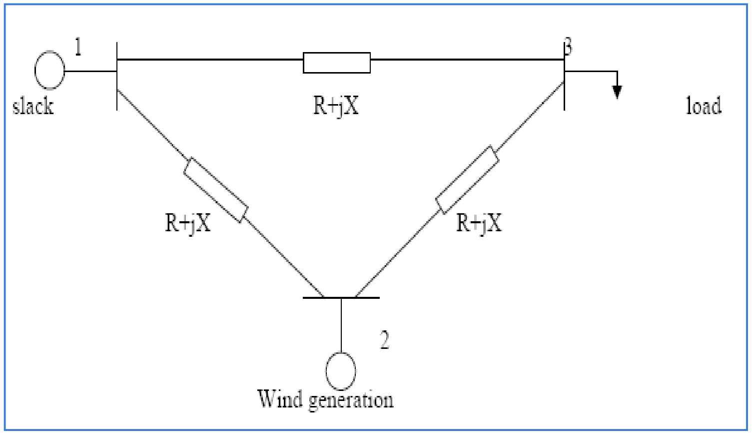
Input basic load flow data and sigma points

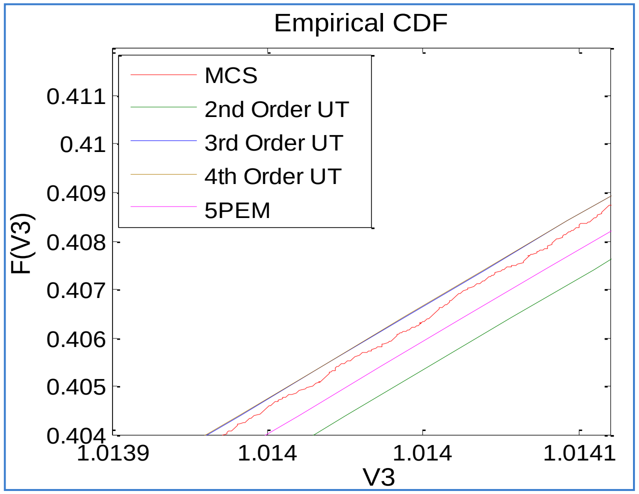
Run load flow using Newton Raphson

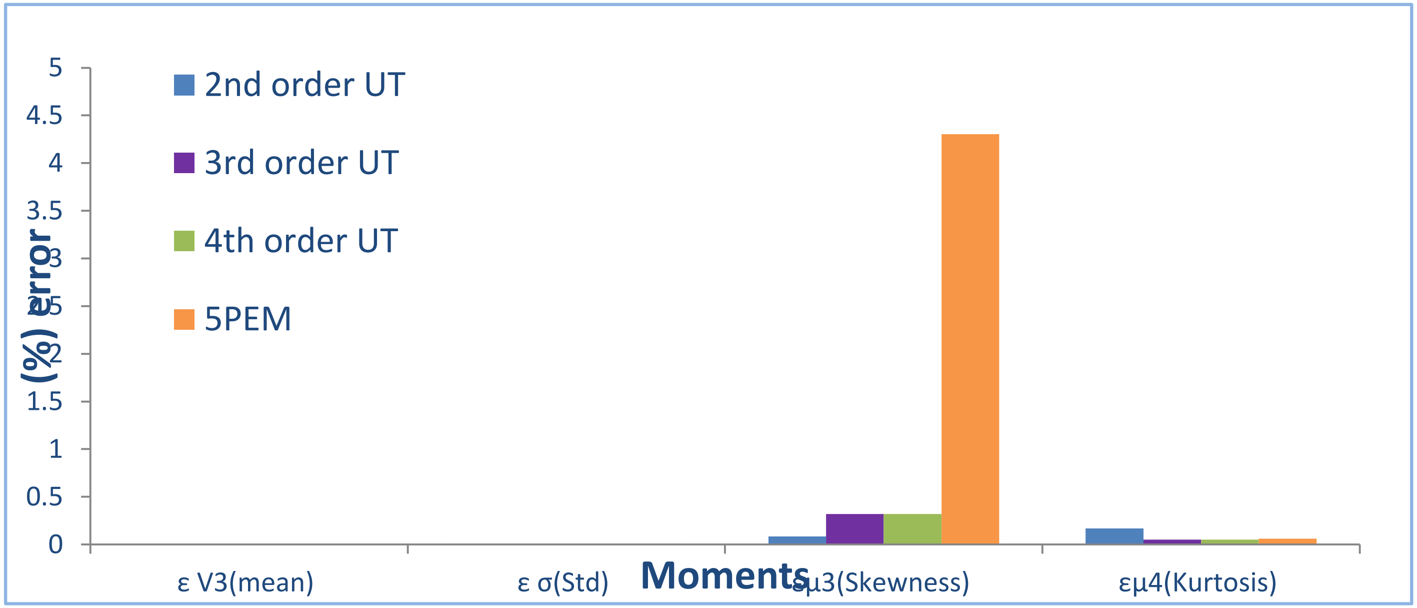
Compute statistical data (moments and cumulants)

Plot CDF for desired variables using statistical data.

Simple 3 Bus Test System

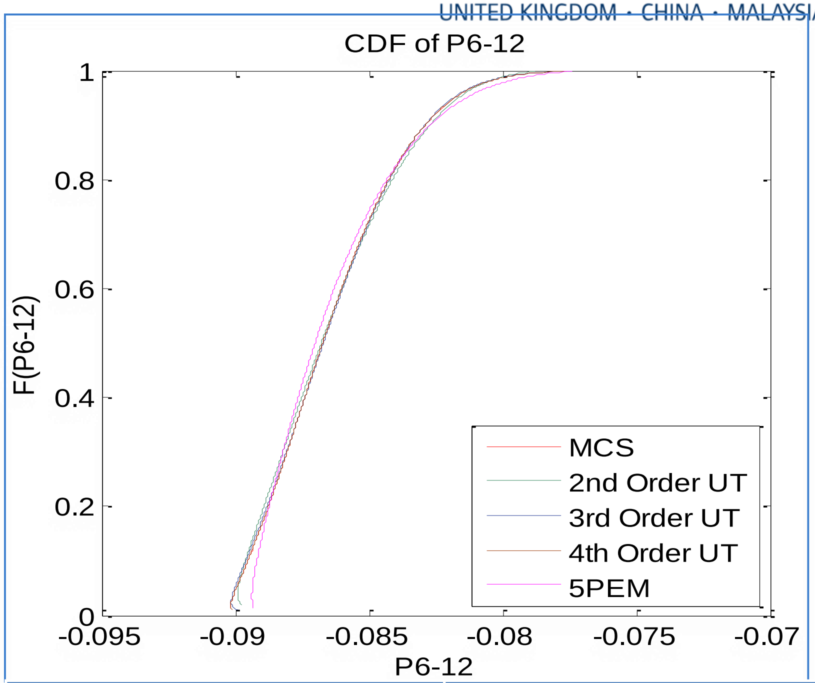




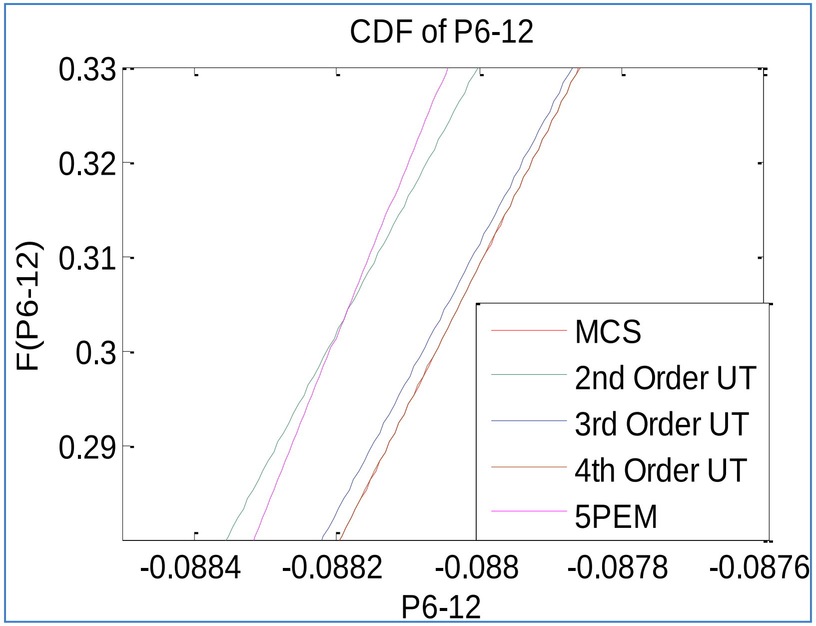


IEEE 14 Bus Test System

- A 50MW rated wind farm on Bus 6
- Varying active and reactive load on Bus 9



Method	Computation Time (S)
2nd Order UT	0.22054
3rd Order UT	0.50433
4th Order UT	0.97112
5PEM	0.9478
MCS	835.574



Summary

- ➢The UT method has been introduced as a method for carrying out PLF.
- ➢The performance of the UT method has been evaluated by comparing results obtained with those from MCS and 5PEM, using a simple 3 bus test system and the IEEE 14 bus test system.
- ❖Correlation between the random variables to be considered.
- ❖Method to be extended for conducted emissions

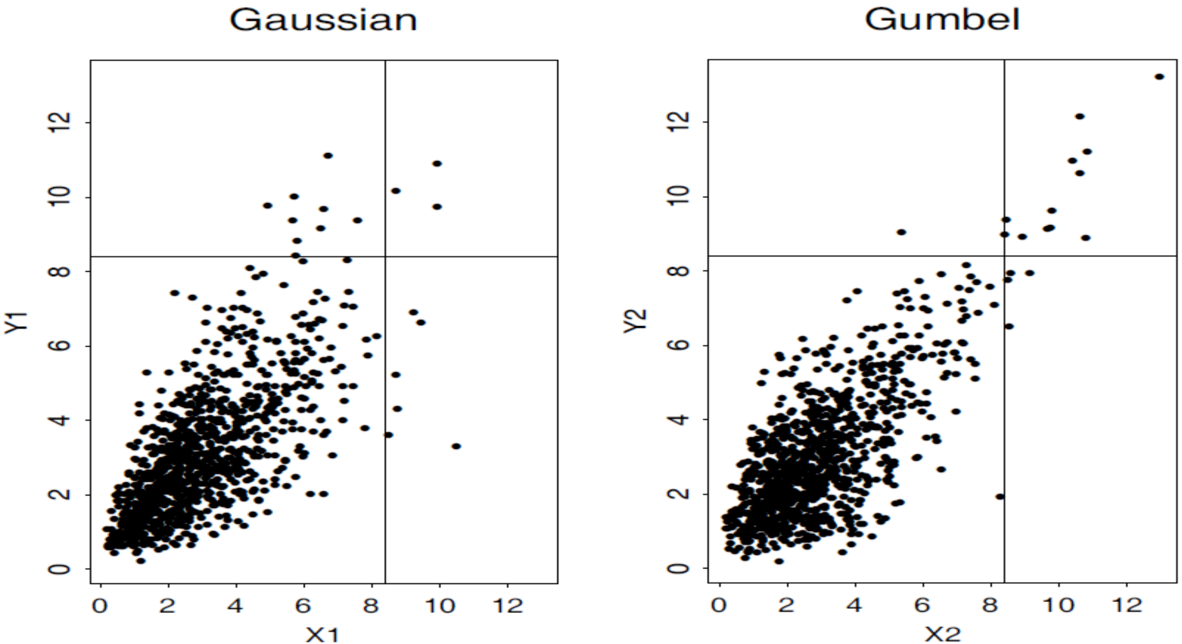
Wind Farm Sites and Dependence

Source: http://www.renewables-map.co.uk

Dependence Versus Correlation

- Dependence measures the statistical relationship between two random variables
- Correlation is the strength of relationship between two or more random variables
- Linear Correlation measures how two random variables are proportional to each other.

- Zero correlation does NOT imply zero dependence



1000 random variates from 2 identical Gamma marginal distributions and identical correlation but different dependence structure

Measures of Dependence

- Pearson Product Moment Correlation coefficient
  - Measures **linear** dependence between variables
  - Simple and commonly used
- Spearman Rank-Order Correlation Coefficient
  - Nonparametric
  - Robust and resistant to data defeats
- Kendall’s Tau
  - Nonparametric
  - Natural
- Blomqvist Beta
  - Nonparametric
  - Fast with low computational complexities

Techniques for Dependent Variable Generation

❖Rosenblatt Transformation

Only applicable to Gaussian distribution

Requires prior full knowledge of random variables joint distribution

❖Nataf Transformation

Based on linear dependence

Transforms correlated variables into standard normal variables

❖Copulas

They join marginal distributions of a set of variables to their joint distribution function

Captures dependence structure of any set of random variables

Copulas

Mathematical Basis

$$\underbrace{F x_1,K\ ,x_n(x_1,\Lambda\ ,x_n)}_{\text{Joint Distribution Function Functions}} = C(\underbrace{F_{X_1}(x_1),F_{X_2}(x_2),\Lambda\ ,F_{X_n}(x_n)}_{\text{Marginal distribution}})$$

Families

❖ Elliptical Copulas

- Easily determined from covariance matrix of the marginal distributions
- Applicable to symmetrical and radial distributions
- Poor for distributions with strong tail dependence
- Examples: Gaussian Copula and Student T copula

❖ Archimedean Copulas

- Flexible and uniquely constructed through a generator
- Dependence parameters other than Pearson rho needed to generate copula function
- Examples: Gumbel copula, Frank copula, Clayton copula

Dependent Variable Generation Using Copulas

Generate W, a (n-by-m) uniformly distributed variables on the interval [0, 1].

Evaluate the partial differential of one of the margins (say u1) with respect to the other margins. For instance for a bivariate Clayton copula with distributions u_1 and u_2;

$$c(u_2) = \left.\frac{\partial C_\alpha(u_1, u_2)}{\partial u_1}\right|_{u_1 = u_{1o}} = u_1^{-(1+\alpha)} \times [u_1^{-\alpha} + u_2^{-\alpha} - 1]^{-(1+\alpha)/\alpha}$$

Substitute u1 the first set of random variables W(1,m) with m random samples into the equation. Let the partial derivative in step 2 above c(u2) equal to W(2,m). With this, u2 can easily be evaluated.

The desired input random variables are then determined using the inverse CDF of their margins.

$$\boldsymbol{X}_i = F_{X_i}{}^{-1} u_i], \qquad i = 1,2$$

Case Study

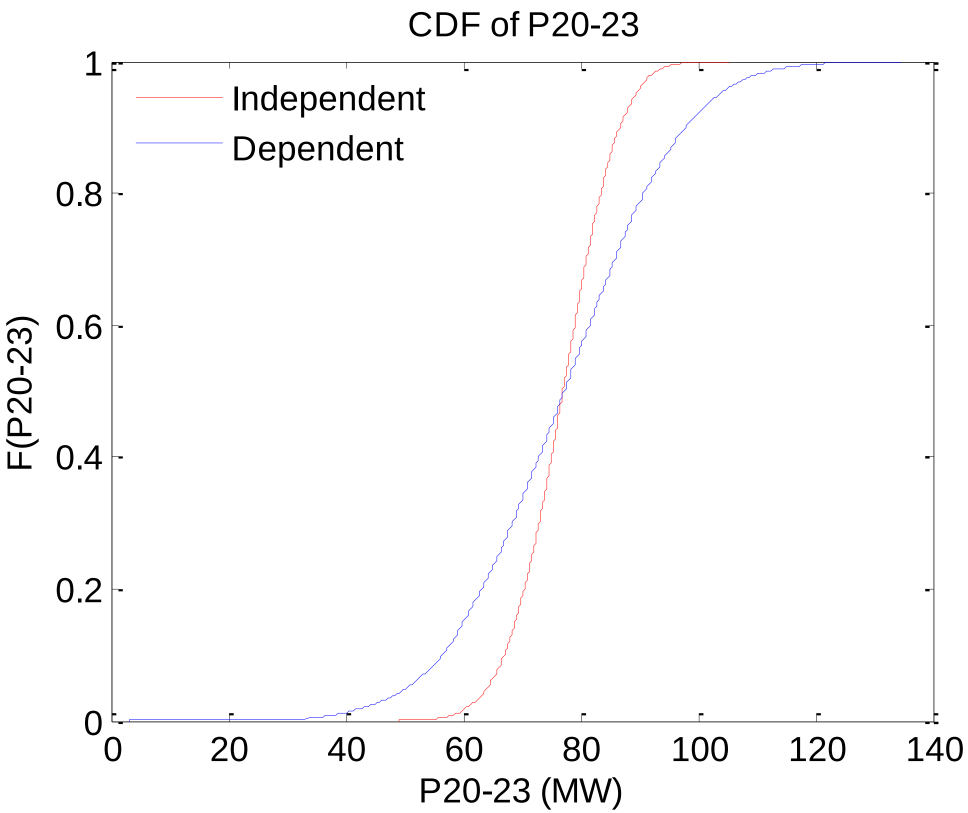
IEEE 24 Bus RTS System with Wind farms

Modifications

- System sectionalized into 2 areas, Area 1= buses 1-13,24; Area 2=buses 14-23
- 2 wind farms in Area 1 located on buses 4 and 9
- Each wind farm made up of an aggregate of 60 2.3MW turbines
- Wind speed parameters: 2.025 and 9 respectively for shape and scale parameters
- Turbine Parameters: 3m/s, 13m/s and 25m/s for cut-in, rated and cut-out wind speed
- 5% coefficient of variation for all active and reactive loads.

Results & Discussion I

CDF of Power Flow Between Buses 20 and 23 for Dependent and Independent Cases



Results & Discussion II

Percentage Variation in Dependent and Independent Moment (relative to Dependent)for the 24 Bus RTS

Moment		Average	Maximum
Voltage	Mean	0.0089	0.0330
	Standard dev	13.920	46.219
	Skewness	56.365	710.34
Angle	Mean	0.5737	1.9500
	Standard dev	41.213	49.309
	Skewness	59.353	112.59
Active Power	Mean	1.7609	28.441
	Standard dev	29.330	52.967
	Skewness	66.252	221.68
Reactive Power	Mean	1.1218	9.6439
	Standard dev	27.745	49.323
	Skewness	119.26	1569.9

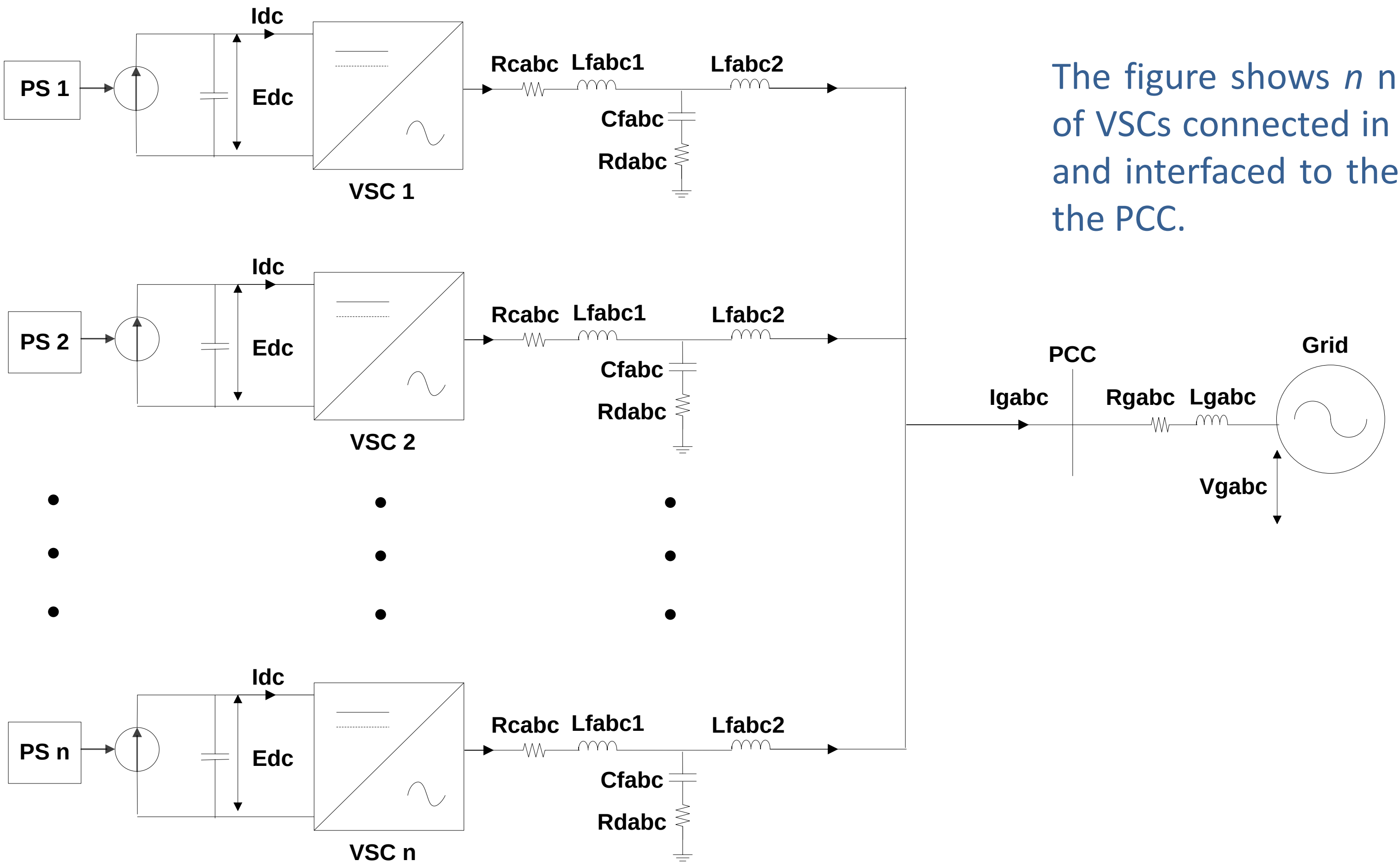
POWER INJECTED AT BUSES 4 AND 9 FOR A 138MW RATED WIND FARM

Method	P Bus4 (MW)	P Bus9 (MW)
Nataf	44.723	44.748
Gumbel	43.795	44.764
Clayton	45.347	45.714

Summary

- Assumption of independence between variables may lead to large errors.
- dependence may exist between variables even when the linear correlation coefficient is zero.
- Other coefficients other than the Pearson moment product are needed for full dependence representation
- Copulas are effective in dependence representation of any variable type.
- Data sample should be analysed to understand its best copula fit.

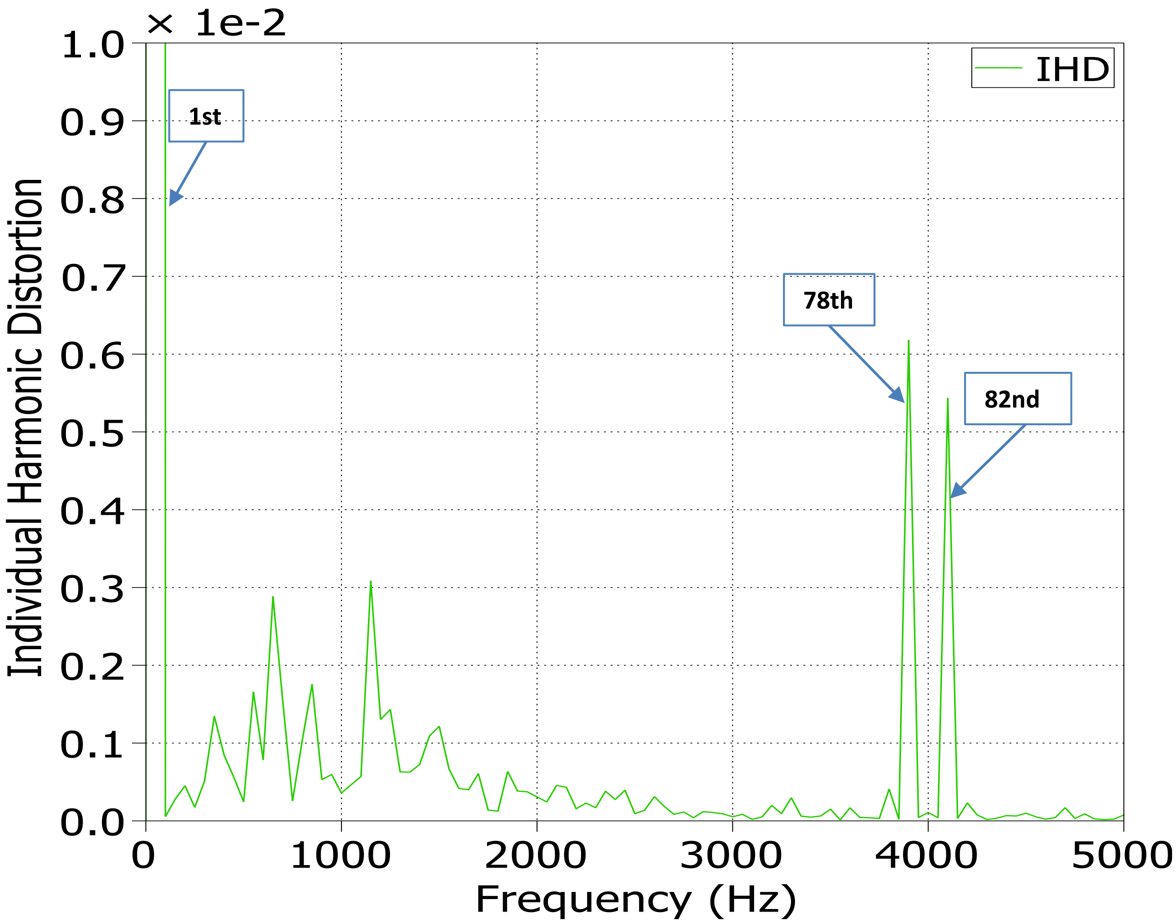
VSC Structure



The figure shows *n* numbers of VSCs connected in parallel and interfaced to the grid at the PCC.

Shows the schematics of the studied VSC system

Conducted Emissions

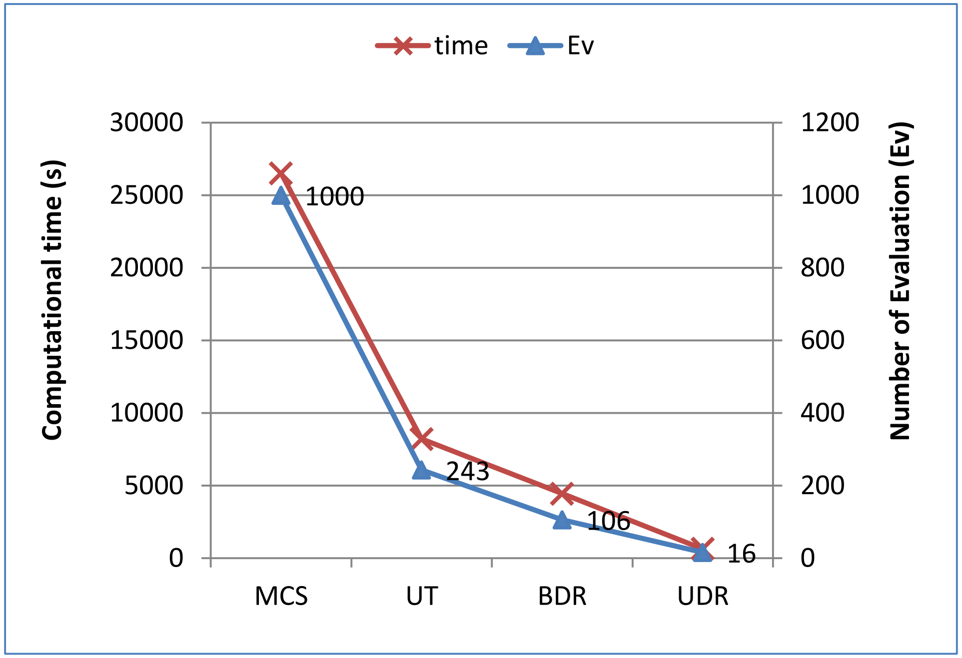


It can be observed from the figure that high emissions are present at the sidebands of the VSC switching frequency (f_{sw} = 4kHz). These emissions are at the 78th and 82nd order.

Shows the conducted emission at sidebands of the switching frequency relative to the fundamental frequency

Simulation

- The number of evaluations and computational time utilized by each method in predicting the conducted emission of 5 VSCs is given below.
- 3 sigma points approximation was used for the UT, BDR and UDR techniques, and 1000 simulations for the MCS approach.

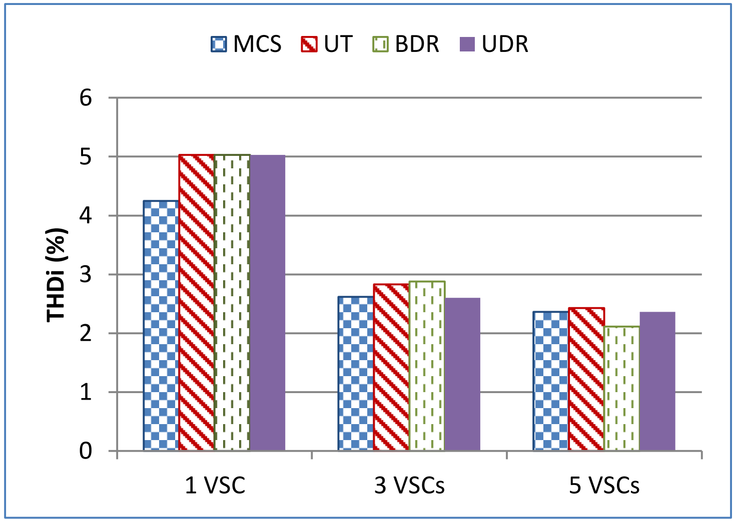


Evaluation Number and Simulation Time for MCS, UT, 3pts BDR and UDR

Power Variation

THDi due to Power Variation

	Mean, μ (%)				Standard deviation, σ			
Method *n*VSC	MCS	UT	BDR	UDR	MCS	UT	BDR	UDR
1	4.25	5.03	5.03	5.03	1.86	2.24	2.24	2.24
3	2.62	2.83	2.88	2.6	0.82	0.78	0.79	0.79
5	2.36	2.43	2.11	2.36	0.27	0.32	0.23	0.20



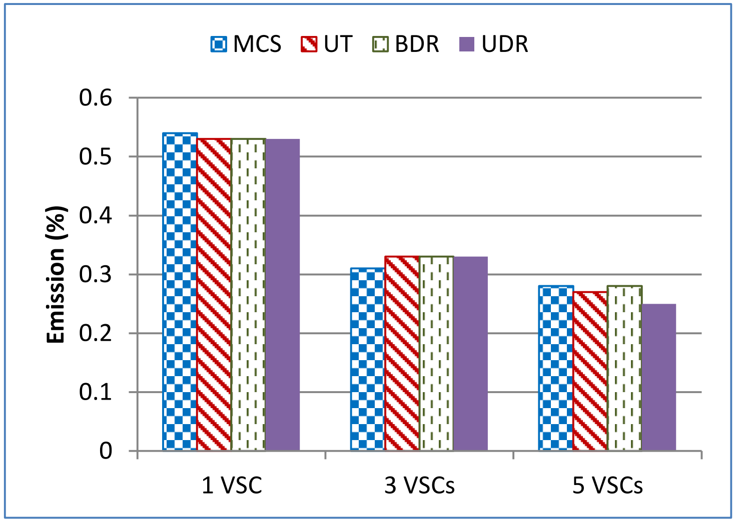
Mean THDi for *n*VSCs under power variation

- THD for 1 VSC is the same using the UT and the dimension reduction methods.
- However, they all over predicted the THD by 15.5% when compared to the MCS.
- The standard deviation values are close to the MCS approach.

Power Variation

78^{TH} order emission due to Power Variation

	Mean, μ (%)				**Standard deviation, σ**			
Method / ***n*VSC**	**MCS**	**UT**	**BDR**	**UDR**	**MCS**	**UT**	**BDR**	**UDR**
1	0.54	0.53	0.53	0.53	0.251	0.248	0.248	0.248
3	0.31	0.33	0.33	0.33	0.055	0.061	0.061	0.052
5	0.28	0.27	0.28	0.25	0.029	0.030	0.032	0.028



Conducted emission mean value at the 78th order under power variation

- Better accuracy was achieved using the BDR and UDR technique in predicting the conducted emissions at the 78th order.
- The standard deviation values are in close agreement with the MCS approach.
- The UDR results are close to the MCS for its mean and standard deviation (Std) only.

Summary

- Conducted emissions of multiple VSCs were investigated and predictions made using the univariate and bivariate dimension reduction method.

- Majority of the predicted conducted emission results showed that the BDR and UDR techniques have a good agreement with the MCS approach.

- BDR and UDR can be used when MCS is not practical in predicting conducted emissions of a large number of VSCs when there are uncertainties in the system or the VSCs

Conclusion

- EUT has been introduced
- Shown to be an efficient and accurate method to estimate stochastic parameters in power systems
- Has been demonstrated for loadflow studies and conducted emission studies

Thank You?