Analysis on Methods to obtain Lumped Parameters for an Inductive Coupling Device

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***Abstract*— A reliable lumped parameters model for an Inductive Coupling Device (ICD) is necessary for many reasons, mainly to develop a robust device. This paper compares an analytical method, a proposed method through Finite Element Method (FEM) and two different physical measurement methods. It shows the advantage of FEM over the analytical method, especially in situations with misalignments. This paper also outlines the two measurement methods using different measurement devices: impedance analyzer and vector network analyzer.**

***Index Terms*—Inductive Power Transmission, Finite Element Methods, Impedance measurement, Inductance measurement**

I. INTRODUCTION

Inductive Coupling Devices (ICDs), such as Wireless Chargers, Transcutaneous Energy Transmitters (TETs) [1], Induction Cookers, etc., are devices which use inductive link as their main principle. They are mainly composed of a primary coil inductively coupled to a secondary coil. In many of these applications, a strong coupling between the coils is difficult to achieve due to the air gap related to misalignments and space between the axes of the coils. In order to reduce their losses, the leakage inductance can be compensated by resonant capacitors [2] and/or the quality factor can be increased. Thus, a good lumped parameters model is necessary to predict their behaviors prior to fabrication in order to develop robust devices.

One way to model the ICD is using the transformer equivalent circuit. It contains parameters to represent the copper losses in the coils (r_1 and r_2) and the magnetic flux behavior with self (L_1, L_2) and mutual (M) inductances. This model is very useful to calculate voltage, current and power with any load without the need to implement a physical device. However, the parameters must be reliable; otherwise, the prediction will be false.

Some researchers use analytical expressions for circular coils with some considerations [3], [4], thus making it invalid for some displacements. Another way to estimate these parameters is through virtual measurements performed by Finite Element Method (FEM) [5]. This method allows calculation of the parameters while considering more complex circuit models.

This paper uses these methods to obtain the ICD model parameters for a TET with two pancake coils and compares the results with experimental data measured by an impedance analyzer (IA) and a vector network analyzer (VNA).

II. METHODOLOGY

This paper considered a TET with 45 and 23 circular and concentric turns respectively in the primary and secondary to compare methods that calculate (analytically and by simulation through FEM) and measure such parameters.

In order to attain the mutual inductance between the coils, the following steps were performed: i) physical measurements by VNA and IA at frequencies between 20 kHz and 1 MHz; ii) analytical and FEM calculations at frequencies of 50, 100, 150, 200, 250 and 300 kHz. These steps were performed under two different scenarios: 1) the coils were aligned and separated by different distances; 2) the coils were separated by 5 mm and their centers were misaligned by different distances.

The self-inductance was obtained by FEM, calculated analytically and measured only by IA because it does not depend on the position of the coils.

A. *Physical Measurements*

The IA supplies data of frequency f in Hz, modulus ($|Z|$) in ohms and phase (φ) in degree. Thus, considering that the circuit can be modeled by an inductor in series with a resistor, the inductance (L) can be calculated by

$$\mathrm{L} = |\mathrm{Z}|.\sin(\varphi.\pi/180)/2.\pi.\mathrm{f}\,. \tag{1}$$

Thus, keeping the coils neither electrically nor magnetically coupled, the self-inductances of each coil were measured independently by the IA for each frequency.

Afterwards, the mutual inductance (M) was obtained by two impedance measurements through the IA: first the coils were coupled in the series opposing (Lo) configuration and second they were in the series non-opposing (La).

$$\mathrm{M} = (La - Lo)/4\,. \tag{2}$$

A second measurement was carried out using a VNA. The Channels 1 and 2 were connected to the primary and secondary coils using current probes, respectively. The primary coil was supplied with the signal from VNA and the secondary was short circuited. The VNA was configured to compute the ratio of the currents of the secondary and of the primary, for a frequency range of 20 kHz to 1 MHz. Considering that the coil resistance is much smaller than its inductance, the mutual inductance can be calculated with the VNA data and the previously measured self-inductance:

$$\mathrm{M} = L.10^{VNA/20}\,. \tag{3}$$

B. Analytic Calculation

Since the coils are circular and concentric, the diameter of each turn increases at least by the wire diameter of the previous turn. Thus, the mutual inductance among all turns can be written as [4]:

$$\mathrm{M} = \sum_{i=1}^{N1}\sum_{j=1}^{N2} M\left(a_i, b_j, \Delta, d\right). \tag{4}$$

In (4), N_1 and N_2 are respectively the number of turns in the primary and secondary coils, a and b are the inner diameters of the primary and secondary coils respectively, Δ is the distance between the central axes of the coils, and d is the distance between the planes of the coils. $M(a_i, b_j, \Delta, d)$ is the mutual inductance between a single turn with inner diameter a_i in the primary and a single turn with inner diameter b_j in the secondary when the coils are misaligned by Δ and separated by d. It can be approximated by [3]:

$$\mathrm{M}\left(a_i, b_j, \Delta, d\right) = \left(\mu_0.\mathrm{a_i}.\mathrm{b_j}\right)/2.\pi.\oint \cos(\beta)/\sqrt{\mathrm{a_i}.\mathrm{b_L}}.G(r).d\theta \tag{5}$$

where

$$\mathrm{r} = \sqrt{\left(4.\mathrm{a_i}.\mathrm{b_L}\right)/\left(\left(\mathrm{a_i}+\mathrm{b_L}\right)^2 + d^2\right)}. \tag{6}$$

$$\mathrm{b_L} = \sqrt{\mathrm{b_j}^2 + \Delta^2 + 2.\Delta.\cos\theta}. \tag{7}$$

$$\tan\beta = \Delta\sin\theta/\left(\mathrm{b_j} + \Delta\cos\theta\right). \tag{8}$$

$$G(r) = (2/r - r).K(r) - 2/r.E(r). \tag{9}$$

Where $K(r)$ and $E(r)$ are the elliptical integral of first and second kind, respectively and θ is the angle of integration going from 0 to $2.\pi$.

The self-inductance is calculated through [4]:

$$\mathrm{L} = L(a_i, r_w) + \sum_{i=1}^{N}\sum_{j=1}^{N} M\left(a_i, b_j, \Delta = 0, d = 0\right). \tag{10}$$

In (9), $L(a, r_w)$ is the inductance of one single turn with inner radius a and wire radius r_w, calculated as

$$\mathrm{L}(a, r_w) = \sum_{i=1}^{N} \mu_0.a.\left(\ln(8.a/r_w) - 2\right). \tag{11}$$

C. Simulation through FEM

The FEM simulates a sinusoidal input signal of arbitrary frequency and amplitude at the primary coil and supplies the voltage and current at the primary (V_1, I_1) and secondary (V_2, I_2) coils with their respective phase. Thus, the FEM simulated the designated coil considering that: i) the secondary has no load, obtaining Vo_1, Io_1, Vo_2 and Io_2; ii) the secondary is short-circuited, obtaining Vcc_1, Icc_1, Vcc_2 and Icc_2. Then, the parameters were calculated by

$$r_1 = real\left(\left(Vo_1 - a.Vo_2\right)/Io_1\right). \tag{12}$$

$$L_1 = imag\left(\left(Vo_1 - a.Vo_2\right)/Io_1\right)/\omega. \tag{13}$$

$$M = -imag\left(Vo_2/Io_1\right)/\omega. \tag{14}$$

$$r_2 = real\left(\left(Vcc_1 - \left(r_1 + j.\omega.L_1\right).Icc_1\right)/a.Icc_2\right). \tag{15}$$

$$L_2 = imag\left(\left(Vcc_1 - \left(r_1 + j.\omega.L_1\right).Icc_1\right)/a.Icc_2\right)/\omega. \tag{16}$$

where $a = N_1/N_2$ is the turn ratio and $\omega = 2.\pi.f$.

III. Results

The comparison between the methods shows that FEM is very accurate for any position of the coils, whereas analytical is valid only for small misalignments. However, the analytical method is very fast when compared to FEM. Fig. 1 presents a plot of the modulus of the mutual inductance vs. misalignment using four methods. The coils were separated by 5 mm.

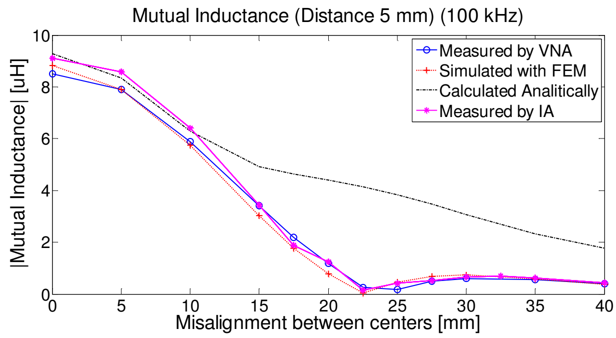

Fig. 1. Modulus of the mutual inductance obtained by the different methods.

Results with the self-inductance and resistance were also obtained showing the advantage of using FEM.

IV. Conclusions

This paper compared an analytical method to obtain lumped parameters of the ICD type TET with a proposed method through FEM. The proposed method showed to be very precise at any condition though slower than the analytical method. The proposed method can be used even in situations where other type of material such as ferrite core is used around the coils whereas the analytical method should be readapted to work in such conditions. Moreover, this paper explained two different methods to measure the mutual inductance. Although the VNA is a powerful tool, it still requires the knowledge of the self-inductance to assess the mutual inductance. The IA measures the mutual inductance without the knowledge of the self-inductance, yet requiring more data at multiple conditions.

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