ADAPTIVE UNSCENTED TRANSFORM FOR UNCERTAINTY QUANTIFICATION IN EMC LARGE-SCALE SYSTEMS

Abstract. The Unscented Transform (UT) is a stochastic collocation method used for uncertainty quantification in nonlinear systems. This methodology is particularly interesting for EMC models with high-fidelity simulations which are time-consuming, since it demands fewer runs of the electromagnetic solver when compared to other classical methodologies. However, since the number of simulations required by the UT increases exponentially with the number of dimensions, this methodology becomes unpractical for large-scale systems. Nevertheless, an adaptive Unscented Transform can be an efficient alternative of uncertainty propagation for these large dimensional systems.

Keywords: Conducted Interference, Electromagnetic Compatibility, Stochastic, Uncertainty, Unscented Transform

INTRODUCTION

Non-intrusive uncertainty quantification methodologies are becoming a popular approach for the uncertainty propagation in complex systems. Many papers recently show the application of stochastic collocation methods or polynomial chaos decomposition in order to estimate the output variance due to parameter uncertainty in many fields of science [1] to [14].

Additionally, the recent development and application of computational tools for electromagnetic compatibility allows accurate simulations of large-scale EMC systems. Many numerical technics of forward modeling in EMC can be mentioned, such as Finite Element Method (FEM), Transmission Line Method (TLM) and Partial Equivalent Element Circuit (PEEC). For instance, a model of a power converter can take into account the intrinsic parasitic effects of components, capacitive and inductive coupling of PCB tracks and nonlinear behavior of semiconductors. Thus, the conducted electromagnetic interference determined using this model is very accurate. However, the computational cost of one accurate simulation is usually very high.

Many parameters of EMC models are actually known up to a certain precision only. This parametric uncertainty is not taken into account by EMC solvers. One possible approach is to develop a nonintrusive methodology, that uses the results of the solvers for different input scenarios and compute statistics of the output. An example of a methodology that has been applied successfully in EMC is the Unscented Transform (UT) [15] to [18].

Although the UT is much more efficient than Monte Carlo simulations to compute the statistical moments (average, mean, skewness and kurtosis), it is not appropriate for large-scale systems. The required number of simulations increases exponentially with the number of dimensions of the model. For instance, a 10-D model would require at least 2808 simulations to estimate the statistical moments of 1 output variable using 4th order UT approximation [16].

In this context, an adaptive UT for the uncertainty quantification of large-scale EMC models that explores the dominance of some dimensions over others is an interesting alternative to classical UT. This novel methodology will be presented and applied to three different large-scale models and compared to Monte Carlo method.

UNSCENTED TRANSFORM

The UT consists of estimating the statistical moments of an output random variable using the result of evaluations of the model on well-chosen input values called sigma points (S_i). A detailed description of the methodology can be found in [15]. The mean ($\overline{G}$) and the variance (σ_G^2), for instance, are given by (1) and (2) respectively.

$$\overline{G} = E\{G(\overline{U} + \hat{u})\} = w_0 G(\overline{U}) + \sum_{i=1}^{N} w_i G(\overline{U} + S_i)\,. \tag{1}$$

$$\sigma_G^2 = E\{(G(\overline{U} + \hat{u}) - \overline{G})^2\} = w_0 (G(\overline{U}) - \overline{G})^2 + \sum_{i=1}^{N} w_i (G(\overline{U} + S_i) - \overline{G})^2\,. \tag{2}$$

where $\overline{U}$ is a vector with average input values, $\hat{u}$ is a vector with zero-mean random input variables, $E\{\}$ is the expectation of a random variable and w_i for $i = 1\ to\ N$ are weights. The expressions for higher-order statistical moments can be found in [15].

There are different ways of determining the sigma points, which correspond to the weights and values of the input parameters where the system must be evaluated. One possible set of sigma points is given by the solution of the system of nonlinear equations in (3) where k is the order of approximation.

$$w_0 = 1 - \sum_i w_i \,, \qquad \sum_i w_i S_i^k = E\left\{\hat{u}^k\right\}, \tag{3}$$

where w_i $(i = 0, \dots, N_{sp})$ are the weights, S_i $(i = 0, \dots, N_{sp})$ are the input values in which the system must be evaluated around average, k is the order of the transform and N_{sp} is the number of sigma points. A sigma point is characterized by its weight and parameter value.

The solution of (3) gives the minimum number of sigma points in order to correctly estimate the output statistical moments and thus it is the computationally cheapest. However, as the number of dimensions of the system increases, the solution of (3) becomes complex.

There's a different set of sigma points which is much simpler to determine and is given by the following expressions [15].

$$S_{i,1} = (\pm 1, \pm 1, \dots, \pm 1)\frac{\sqrt{N_{rv}+2}}{\sqrt{N_{rv}}}\boldsymbol{\sigma}, \quad w_1 = \frac{\boldsymbol{N_{rv}^2}}{\boldsymbol{2^{N_{rv}}(N_{rv}+2)^2}} \,,$$

$$\begin{aligned} S_{i,2} &= (\pm 1, 0, \dots, 0)\sqrt{N_{rv}+2}\,\boldsymbol{\sigma}, \\ S_{i,2} &= (0, \pm 1, \dots, 0)\sqrt{N_{rv}+2}\,\boldsymbol{\sigma}, \quad w_2 = \frac{\boldsymbol{1}}{\boldsymbol{(N_{rv}+2)^2}} \,, \\ &\dots \\ S_{i,2} &= (0, 0, \dots, \pm 1)\sqrt{N_{rv}+2}\,\boldsymbol{\sigma}, \end{aligned} \tag{4}$$

where N_{rv} is the number of random variables and $\boldsymbol{\sigma}$ is the vector with the input standard deviations. The set of sigma points given by (4) eliminates the need to solve a highly complicated system of nonlinear equations but it requires more evaluations of the system. Table 1 shows numerical values of w_i and S_i from (4) for problems with a number of random variables from 1 to 5, with 3 digits precision.

Table 1. Sigma points and weights given by (4) with $\sigma = 1$ and $N_{rv} = 1 \dots 5$.

N_{rv}	w_1	$S_{i,1}$	w_2	$S_{i,2}$
1	0.056	(± 1)*1.732	0.111	(± 1)*1.732
2	0.063	$(\pm 1, \pm 1)$*1.414	0.063	$\{(0, \pm 1)$ and $(\pm 1, 0)\}$*2.000
3	0.045	$(\pm 1, \pm 1, \dots, \pm 1)$ $*$1.291	0.040	$(0, \dots, \pm 1, \dots, 0)$*2.236
4	0.028	$(\pm 1, \pm 1, \dots, \pm 1)$ $*$1.225	0.028	$(0, \dots, \pm 1, \dots, 0)$*2.449
5	0.016	$(\pm 1, \pm 1, \dots, \pm 1)$ $*$1.183	0.020	$(0, \dots, \pm 1, \dots, 0)$*2.646

Thus the UT consists briefly of calculating a set of sigma points by (3) or (4), evaluating the system at this set and applying (1) and (2) to obtain the average and standard deviation of the output variable. The only issue about the methodology is for high-dimensional systems, since the number of sigma points increases very fast with the dimensionality.

ADAPTIVE UNSCENTED TRANSFORM

An alternative method to the classical UT is described as follows: rank the input parameters by their influence on the output variable, apply the UT considering only the most important variable and successively one more variable at a time, following the order of importance and stop when convergence is reached. In a large-scale system that only a few input parameters must be considered, this adaptive approach is very effective. Figure 1 presents an overview of the adaptive UT.

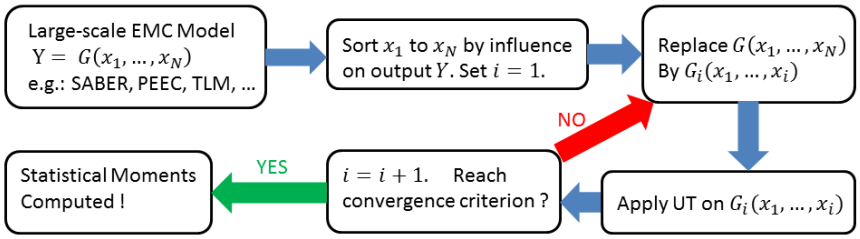

Figure 1. Overview of adaptive Unscented Transform

There are several approaches in order to classify the importance of input parameters on a given output variable, for instance [19] and [20]. In this paper, we consider the simplest way to estimate, which is based on the partial derivative of the output with respect to each input and on the amount of uncertainty of each input. Thus, the expression is:

$$rank_i = \left|\frac{\partial G}{\partial x_i}\right| * \sigma_i \ , \tag{5}$$

where $rank_i$ is the importance of the i-th input variable, $\left|\frac{\partial G}{\partial x_i}\right|$ is the partial derivative of the system G with respect to the i-th input and σ_i is the i-th standard deviation. In this manner, both the derivative and the amount of uncertainty are taken into account. For instance, an input variable with a strong influence on the output but with very low uncertainty or an input variable with weak influence on the output and strong uncertainty may be low ranked compared to an input variable with medium influence and uncertainty.

The partial derivative is computed by the approximate expression given below.

$$\left|\frac{\partial G}{\partial x_i}\right| \approx \left|\frac{G(x_0) - G(x_0 + \Delta x_i)}{\Delta x_i}\right| \ , \tag{6}$$

where x_0 is the vector of nominal parameters and Δx_i is a small variation of the ith parameter. In general, the value of Δx_i does not have a significant impact on the ranking procedure, as it is shown in the results section for model 1.

The convergence criterion is computed through the variance of the response for the mean and standard deviation, as follows:

$$\begin{aligned} &var\{mean\ of\ last\ N\ samples\} < \varepsilon, \\ &var\{var\ of\ last\ N\ samples\} < \varepsilon, \end{aligned} \tag{7}$$

where $var\{\}$ is the variance of a random variable, $mean\{\}$ is the average of a random variable, N is an arbitrary positive integer and ε is a given tolerance. In this paper, the parameters N and ε were fixed to 500 and 0.001, as these values must be chosen by the user depending on the required accuracy.

In other words, the convergence is reached when the variance of the average value of the output is less than a given tolerance and the variance of the standard deviation is less than another given tolerance. This is a much fairer comparison with Monte Carlo. Many papers set a high number of simulations for Monte Carlo method without checking the convergence.

The adaptive UT has been implemented and tested in three different large-scale models. The first two models are analytical formulas with different kinds of non-linearity and dimensionality. The third model is a relatively high-fidelity model of a DC-DC Converter which takes into account many parasitic effects and imperfections.

RESULTS

Model 1 – Quadratic polynomial

The first model to be analyzed is given by the following expression,

$$G(\vec{x}) = (1 + \sum_{i=1}^{Nrv} a_i x_i^2) \ , \tag{8}$$

where $N_{rv} = 100$, x_i $(i = 1 \dots N_{rv})$ follow independent normal Probability Density Functions (PDF) with 5% and 15% of standard deviation relatively to the average and the coefficients a_i $(i = 1 \dots N_{rv})$ are chosen so that there are 5 dominant input variables. Figure 2 presents the relative effect of all input variables on the output given by (5), with a $\Delta x_i = 0.1$ of the interval size, for $i = 1 \dots N_{rv}$ while Figure 3 presents the effect of changing the value of Δx_i in the ranking result, for parameters from number 60 to 82. Notice, in Figure 3, that parameters 75 and 77, for instance, have higher ranking than other parameters, no matter the Δx_i chosen.

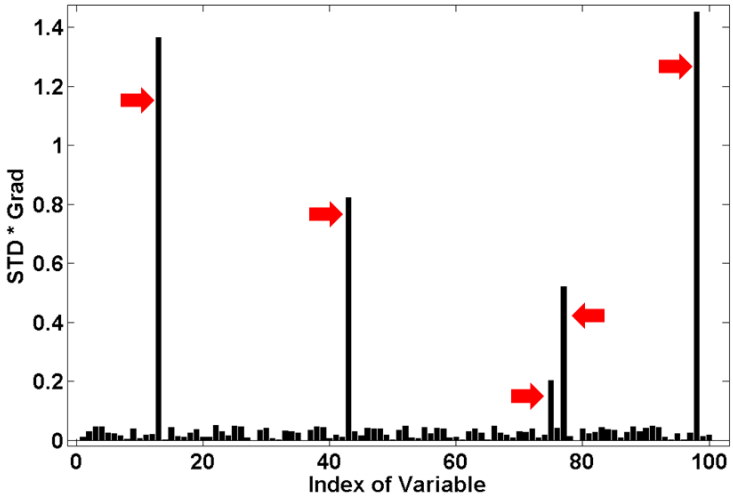

Figure 2. Model 1 – Ranking input variables

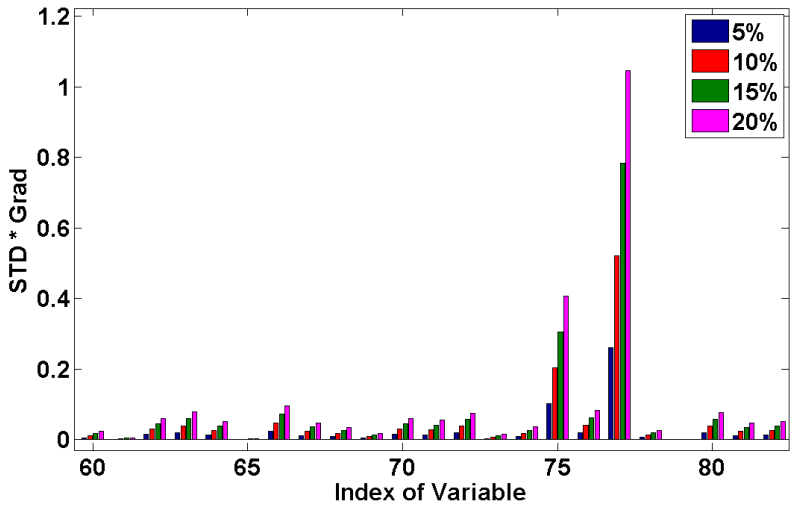

Figure 3 – Comparison of Δx_i (5%, 10%, 15% and 20% of interval width) on ranking result

The coefficients a_i are randomly chosen to produce a generic model but the average of 5 coefficients is 30 times the average of the rest of the coefficients. In this manner, a model with a subset of input dominant variables is created and shown by red arrows in Figure 2.

The results of Monte Carlo method and adaptive UT are presented and compared in Figure 4 and 5, for the two scenario of uncertainty.

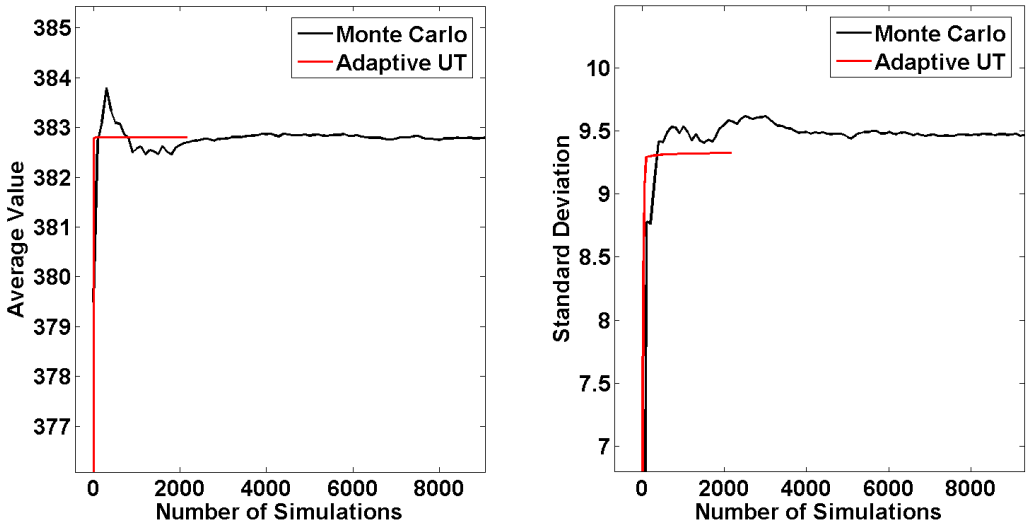

Figure 4. Results for Model 1, 5% Standard Deviation

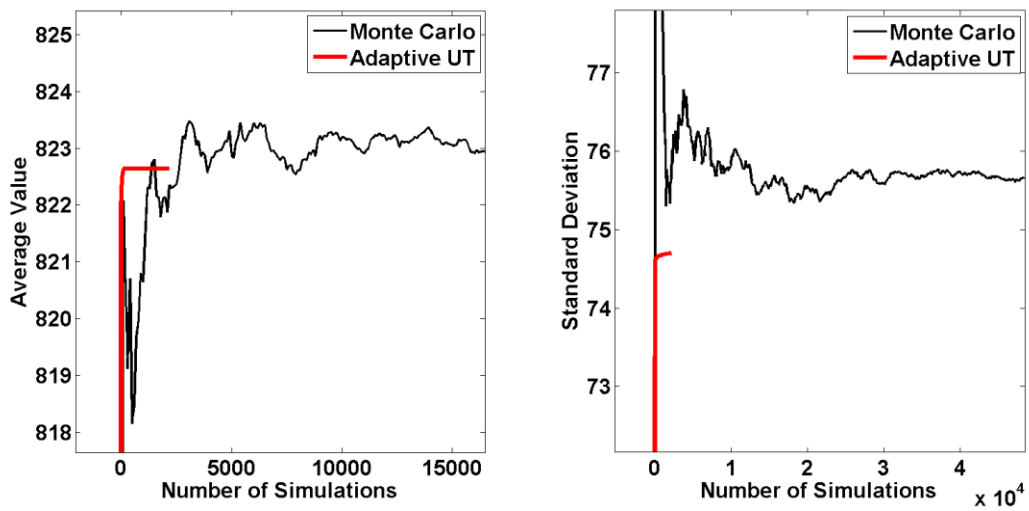

Figure 5. Results for Model 1, 15% Standard Deviation

Figure 4 and 5 show the rapid convergence of the adaptive UT when computing the average and the standard deviation of the output.

Table 2. Comparison between Monte Carlo and Adaptive Unscented Transform – Model 1

Input Uncertainty	Methodology	Mean	Standard Deviation	# solver calls
5%	Monte Carlo	382.8046	9.5272	44200
5%	**Adaptive UT**	382.8015	9.3268	444
15%	Monte Carlo	823.8560	75.5047	200000
15%	**Adaptive UT**	822.6413	74.6990	444

Model 2 – Logarithm, 4th power and inner product

The second model to be analyzed is given by the following expression,

(9) $$G(\vec{x}) = 20\log_{10}\left(\sum_{i=1}^{Nrv} a_i x_i + \sum_{i=1}^{Nrv} b_i x_i^{\,2} + \sum_{i=1}^{Nrv} c_i x_i^{\,3} + \sum_{i=1}^{Nrv} d_i x_i^{\,4} + e_i \prod_{i=1}^{7} x_i + e_i \prod_{i=9}^{Nrv} x_i\right),$$

where $N_{rv} = 200$, x_i $(i = 1 \dots N_{rv})$ follow independent uniform PDF and the coefficients a_i $(i = 1 \dots N_{rv})$ are chosen so that there are 8 dominant input variables. Figure 6 presents the relative effect of all input variables on the output given by (5).

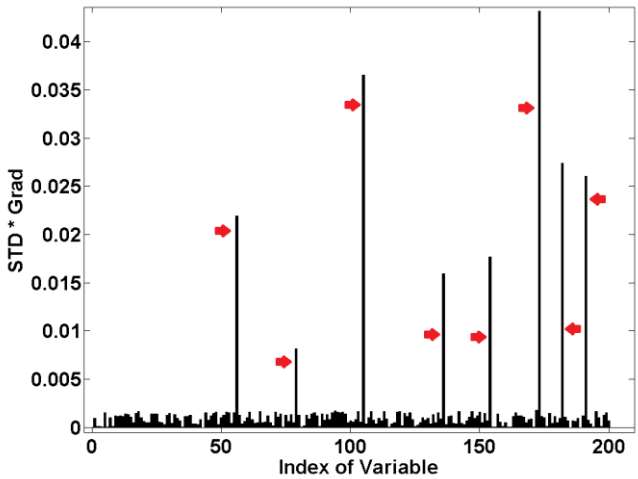

Figure 6. Model 2 – Ranking input variables

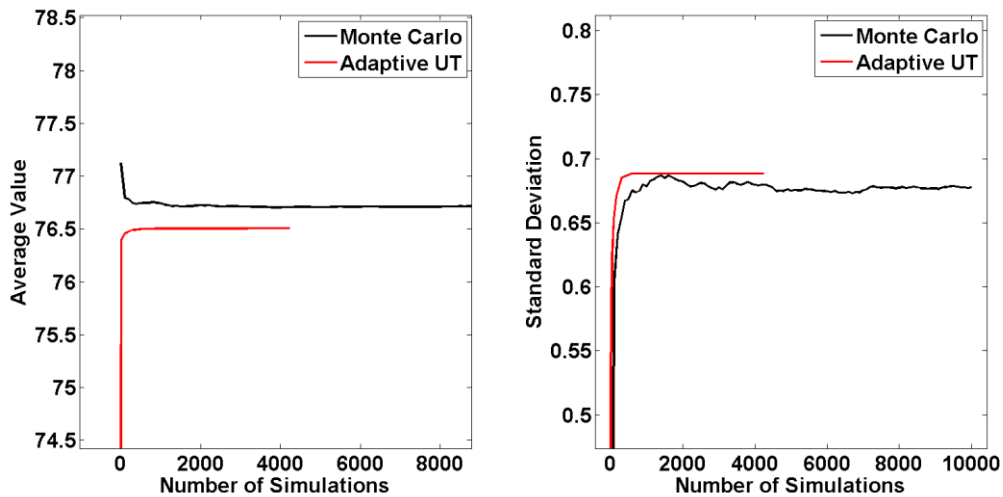

Figure 7. Results for Model 2, 20% interval

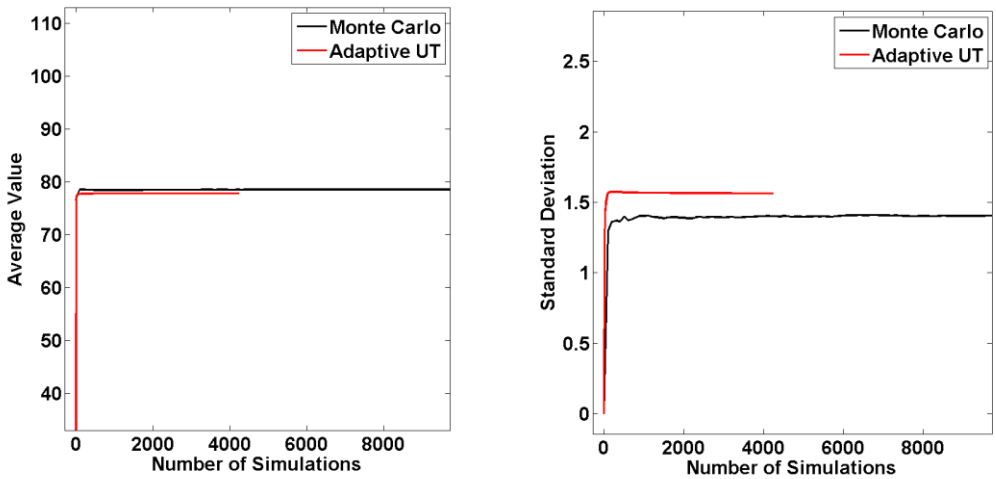

Figure 8. Results for Model 2, 40% interval

Table 3. Comparison between Monte Carlo and Adaptive Unscented Transform – Model 2

Input Uncertainty	Methodology	Mean	Standard Deviation	# solver calls
20%	Monte Carlo	76.7156	0.6776	10000
20%	**Adaptive UT**	76.5069	0.6882	2557
40%	Monte Carlo	78.4996	1.4039	10000
40%	**Adaptive UT**	77.7902	1.5621	2557

Model 3

The third model to be analyzed is a model of a DC-DC Power Converter. Its schematic is shown in Figure 9.

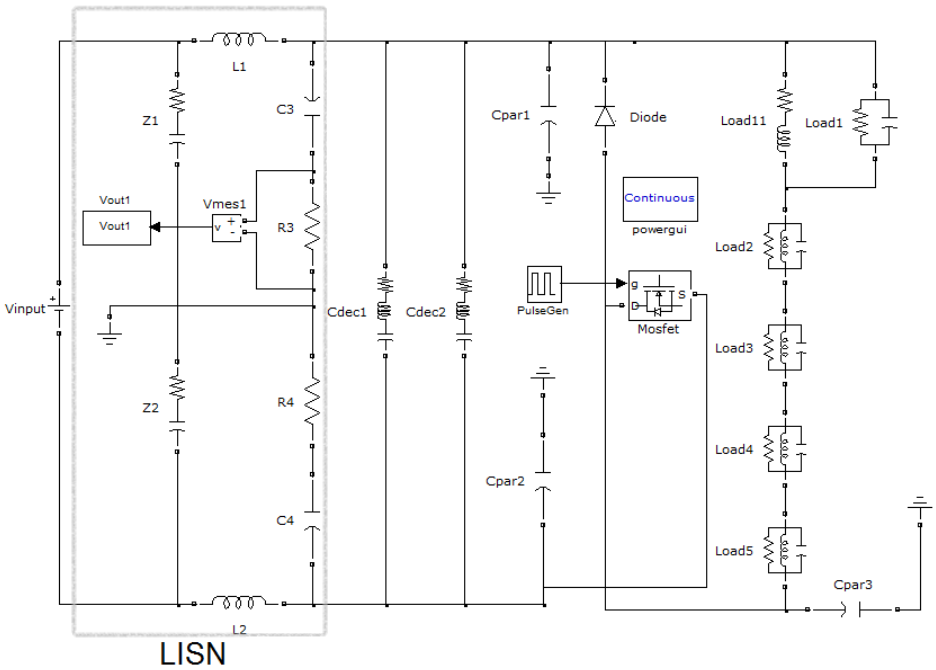

Figure 9. Model 3– Power Converter schematic

The input variables of this model are the voltage source, resistances, capacitances, inductances and semiconductor parameters of the diode and MOSFET. The output variable is the FFT of the voltage across the resistor R_3 in dB at 20kHz in the Line Impedance Stabilization Network (LISN). This output is a measure of the maximum conducted EMI. Thus, this problem is an example of a parametric uncertainty study of a power converter for the assessment

The characteristics of the model are given as follows: $N_{rv} = 45$, x_i $(i = 1 \dots N_{rv})$ follow independent uniform PDF and the coefficients a_i $(i = 1 \dots N_{rv})$ are chosen so that there are 8 dominant input variables. Figure 10 presents the relative effect of all input variables on the output given by (5).

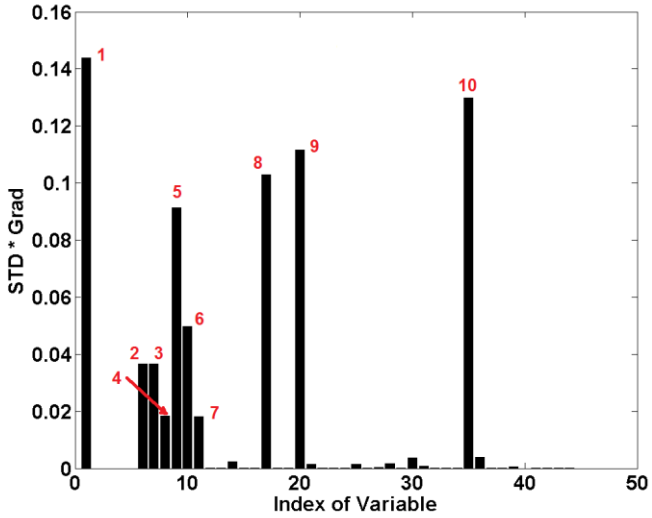

Figure 10. Model 3 – Sensitivity Analysis

The relevant input variables of Figure 10 are described in more detail in Table 4. It can be seen that the parasitic effects of the converter have low impact on the conducted EMI at 20kHz, when compared to the nominal component values.

Table 4. List of Important Components

Index	Parameter	Mean	Index	Parameter	Mean
1	Input Voltage	200 V	6	Resistance R_4	50 Ω
2	Inductance L_1	47.68 µH	7	Capacitance C_4	270.65 nF
3	Inductance L_2	47.68 µH	8	C. decoupling C_{dec1}	899.35 nF
4	Capacitance C_3	270.65 nF	9	C. decoupling C_{dec2}	899.35 nF
5	Resistance R_3	50 Ω	10	Load resistance R_{11}	132.97 Ω

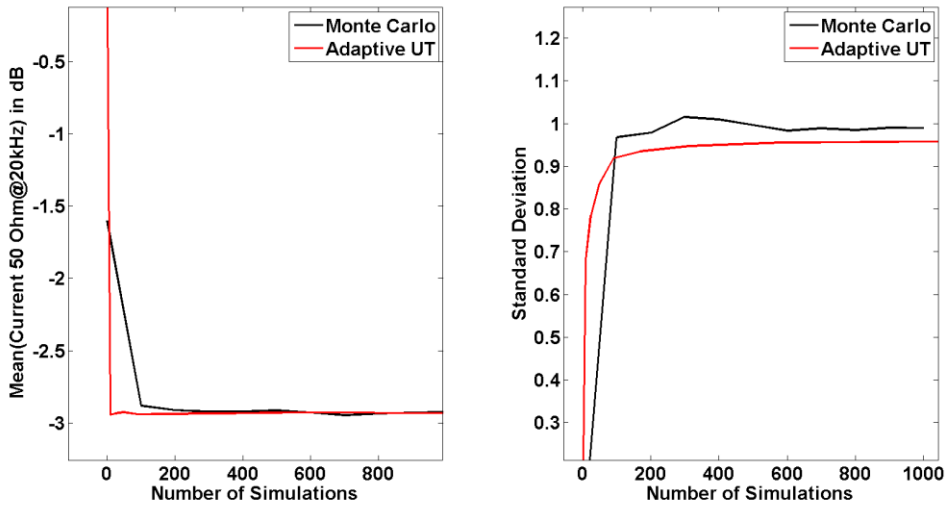

Figure 11. Results for Model 3, 10% interval

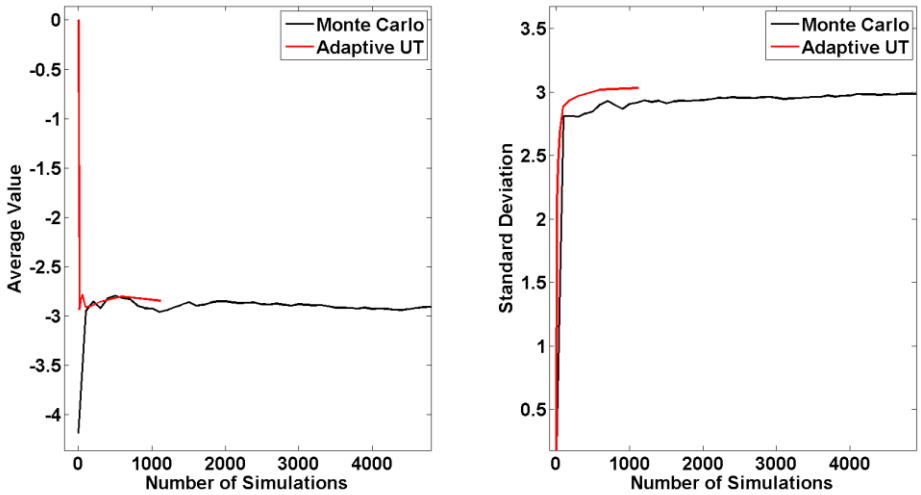

Figure 12. Results for Model 3, 30% interval

Table 5. Comparison between Monte Carlo and Adaptive Unscented Transform – Model 3

Input Uncertainty	Methodology	Mean (dB)	Standard Deviation	# solver calls
10%	Monte Carlo	-2.9224	0.9895	1000
10%	**Adaptive UT**	-2.9371	0.9605	389
30%	Monte Carlo	-2.9169	2.9849	>5000
30%	**Adaptive UT**	-2.8436	3.0328	701

The results presented in Figure 11 show that the convergence of the adaptive UT for the average value was achieved considerably faster than traditional Monte Carlo method. In Figure 12, the improvement brought by the adaptive UT is clear, especially for the assessment of the standard deviation. The results of Table 5 show good agreement between the two methodologies and encourage the further study of the adaptive UT.

CONCLUSIONS

An adaptive collocation method has been presented and successfully tested in three different large-scale models, whereas most of the traditional methodologies for uncertainty quantification can be unfeasible for high-dimensional or high-computational cost models. It showed a much faster convergence rate than Monte Carlo method for similar accuracy. Moreover, the comparison with Monte Carlo was made for a given accuracy, thus being fairer than in other papers. Generally, one sets a very high number of simulations for the Monte Carlo whereas the convergence was reached much earlier.

The first two models were analytical and used to illustrate the general idea of the methodology. The third model was a DC-DC Converter with uncertainty in all its parameters. The adaptive UT turned out to be a good alternative for a fast assessment of the average and standard deviation of the conducted EMI.

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